Homework 1

Due: Tuesday March 24, before class

Question 1

Suppose an agent consumes three goods, $\{x_1, x_2, x_3\}$. Her utility is

$$x_1^2 + x_2^2 + x_3^2$$

The agent has income $M = 1$.

1.1

Suppose the prices are $p_1 = 1$, $p_2 = 1$ and $p_3 = 1$. Solve for the agent's optimal consumption.

1.2

Suppose the prices are $p_1 = 3$, $p_2 = 1$ and $p_3 = 1$. Solve for the agent's optimal consumption.

Question 2

2.1

Show that the CES function $\alpha\frac{x^\delta}{\delta} + \beta\frac{y^\delta}{\delta}$ is homothetic. How does the MRS depend on the ratio?

2.2

Show that your results from part 2.1 agree with our discussion of the cases $\delta = 1$ (perfect substitutes) and $\delta = 0$ (Cobb-Douglas).

2.3

Show that the MRS is strictly diminishing for all values of δ.

2.4

Show that if $x = y$, the MRS for this function depends only on the relative sizes of α and β.

2.5

Calculate the MRS for this function when $y/x = 0.9$ and $y/x = 1.1$ for the two cases $\delta = 0.5$ and $\delta = -1$. What do you conclude about the extent to which the MRS changes in the vicinity of $x = y$? How would you interpret this geometrically?

Question 3

In this problem, we will use a more standard form of the CES utility function to derive indirect utility and expenditure functions. Suppose utility is given by

$$U(x, y) = (x^\delta + y^\delta)^{1/\delta}$$

3.1

Show that in this function the elasticity of substitution $\sigma = 1/(1-\delta)$ and briefly explain the economic interpretation (Hint: Elasticity of substitution $= -\frac{d\ln(x/y)}{d\ln(MU_x/MU_y)}$).

3.2

Show that the indirect utility function for the utility function just given is

$$V = I(p_x^r + p_y^r)^{-1/r}$$

where $r = \delta/(\delta - 1) = 1 - \sigma$.

3.3

Show that the function derived in part 3.2 is homogeneous of degree zero in prices and income.

3.4

Show that this function is strictly increasing in income.

3.5

Show that this function is strictly decreasing in any price.

Question 4

Consider a consumer with preferences over two goods x and y represented by the Stone-Geary utility function:

$$U(x, y) = (x - \gamma_x)^\alpha (y - \gamma_y)^\beta$$

where $\alpha, \beta > 0$ with $\alpha + \beta = 1$, and $\gamma_x, \gamma_y \geq 0$ are called subsistence levels. The consumer faces budget constraint $p_x x + p_y y = I$ and we assume $I > p_x \gamma_x + p_y \gamma_y$.

4.1

Write down the Lagrangian for the utility maximization problem. Derive the first-order conditions and show that at the optimum:

$$\frac{x - \gamma_x}{y - \gamma_y} = \frac{\alpha p_y}{\beta p_x}$$

4.2

Using the result from part 4.1 and the budget constraint, solve for the Marshallian demand functions x^* and y^*. Show that they can be written as:

$$x^* = \gamma_x + \frac{\alpha}{p_x}(I - p_x \gamma_x - p_y \gamma_y)$$
$$y^* = \gamma_y + \frac{\beta}{p_y}(I - p_x \gamma_x - p_y \gamma_y)$$

This system is known as the Linear Expenditure System (LES).

4.3

Provide an economic interpretation of the demand functions in part 4.2. What is the role of γ_x and γ_y? What does the quantity $I - p_x\gamma_x - p_y\gamma_y$ represent, and why does the assumption $I > p_x\gamma_x + p_y\gamma_y$ matter?

4.4

Show that when $\gamma_x = \gamma_y = 0$, the Stone-Geary function reduces to a Cobb-Douglas utility function. Verify that the demand functions from part 4.2 are consistent with this special case.

4.5

Show that the demand functions x^* and y^* are homogeneous of degree zero in (p_x, p_y, I).

4.6

Suppose $\alpha = 0.6$, $\beta = 0.4$, $\gamma_x = 2$, $\gamma_y = 1$, $p_x = 4$, $p_y = 2$, and $I = 30$. Calculate the optimal demands x^* and y^* and verify that they satisfy the budget constraint.